

Control of Induction motor

Introduction :-

The electrical machine that converts electrical energy and mechanical energy into electrical energy. Drive system are mainly used in applications such as pumps, fans, paper and textile mills, electric vehicle and subway transportation, elevators, servos and robotics. Home applications are generally classified into constant - speed and variable speed drives. AC machines have been used in constant - speed application, because conventional methods of their speed control have either been expensive or highly inefficient. But DC machines were preferred for variable speed drives. However, the main disadvantages of DC machines are.

1. Higher cost
2. Higher rotor inertia
3. Maintenance problems with commutators and brushes
4. EMI problems.
5. Do not permit a machine to operate in dirty and explosive environments.

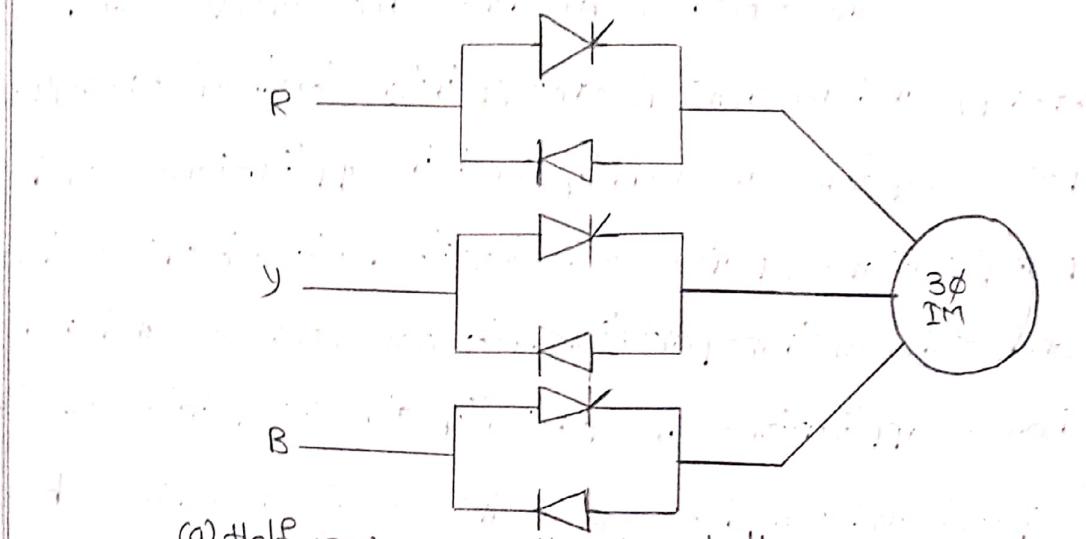
AC Voltage Controller for 3-phase induction motor:-

The stator voltage is controlled in these speed control systems, by means of a power electronic controller. There are two methods of control as follows (a) on-off control (b) phase control.

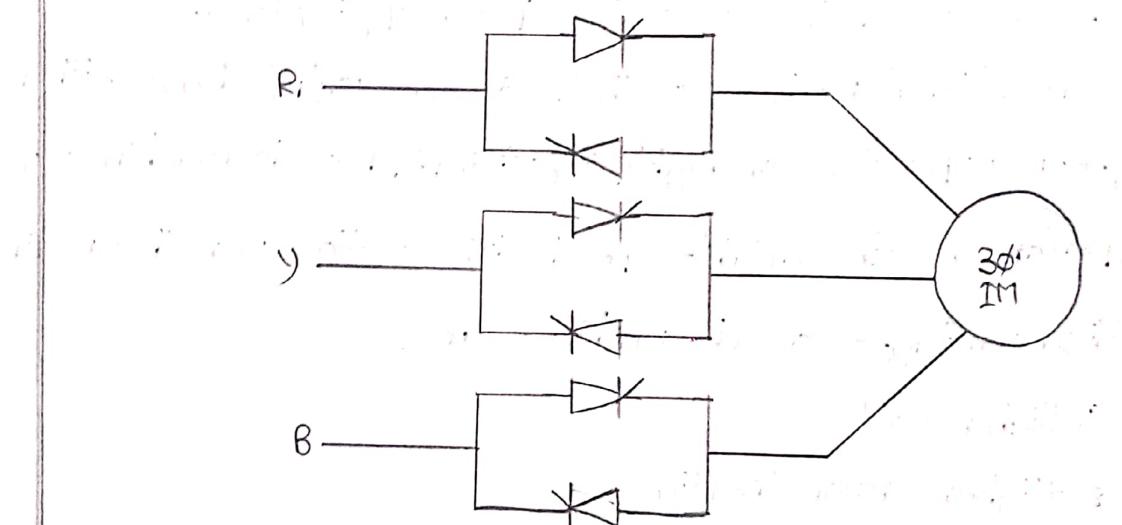
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In on-off control, the transistors are employed as

switches to connect the load circuit to the source for a few cycles. Here thyristors acts as high speed switch (contactor). This method is known as integral cycle control.



(a) Half wave ac voltage controller.



(b) full wave ac voltage controller.

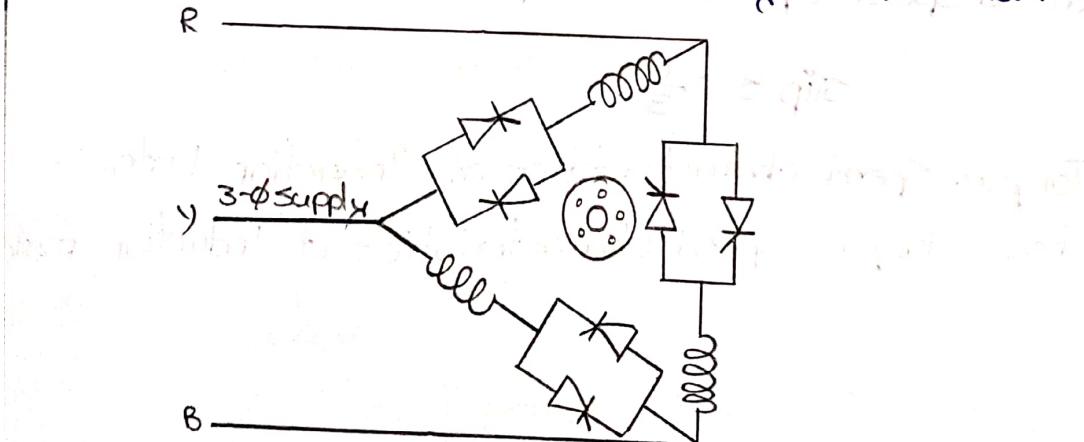
In phase control, the thyristors are employed as switches to connect the load to the ac source for a portion of each cycle of input voltage. The power circuit configuration for on-off control and phase control do not differ in any manner. Normally, thyristors in phase control modes are used. The various schemes are i) single phase or 3 phase half wave ac voltage controller ii) 1φ or 3φ full wave ac voltage controller.

Figure (a) and (b) shows the circuits of three phase half wave and full wave ac voltage controllers.

connected to stators. In half wave ac voltage controller consists of 3SCRs and 3diodes. Here one SCR and one diode in antiparallel are connected between the line and motor in a phase.

The full wave ac voltage controller consists of 6SCRs. Here two SCRs in antiparallel are connected b/w the line and motor in a phase. The main advantage of half wave controller is a saving the cost of semiconductor device and does not give rise to dc components in any part of the system. The disadvantage is that, it introduces more harmonics into the line current. The effective load voltage in three phase ac circuit can be varied by varying the thyristor firing angles.

figure (2) shows 3 phase full wave ac voltage control for delta connected load. It may be used and has the advantage of reducing the current of the device. When the motor is delta connected, the third harmonic voltages produced by motor back emf causes circulating current through the windings which increases losses and thermal loading of the motor.



fig(2) : Three phase full wave ac voltage controller for Delta. For low power rating motors anti-parallel SCR pair can be replaced by a triac. It is shown in figure. AC voltage controllers are also used for soft start of motors.

The Thyristor controller brings in two more sources of power losses. Power loss takes places in the power devices in the controller. In addition, harmonic losses takes place in the motor due to harmonic current flowing in the winding due to phase control. These two additional loss components will make this speed controller further inefficient. Also over heating of the motor on harmonic losses is another possibility. Harmonic currents can result in cogging/crawling etc, especially when attempts are made to turn the motor at very low speeds. The input power factor is also very low.

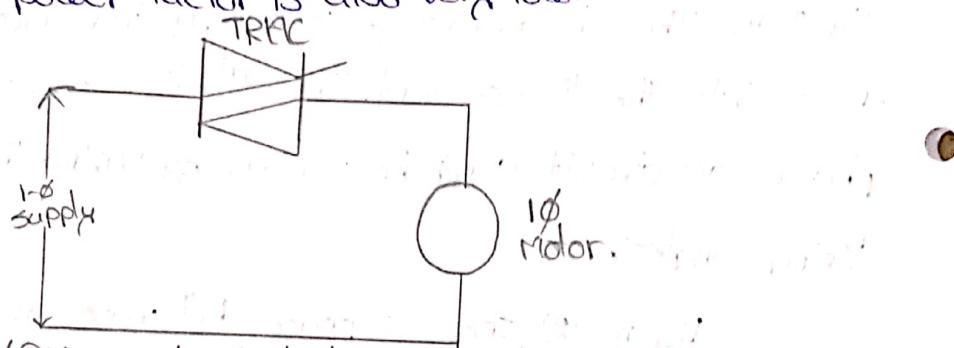
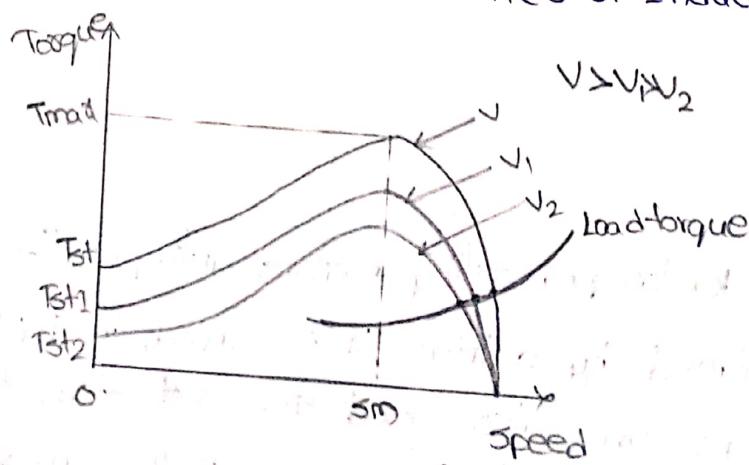


Fig: - 1- ϕ IM speed control by using TRIAC.

In spite of all these problems these speed controllers are popular, especially for fan, pump and crane drives with limited speed variation, due to simplicity and reliability. For these type of loads, the load torque is directly proportional to speed squared and input current is maximum when

$$\text{Slip } s = \frac{1}{3}$$

Torque-Speed characteristics of Induction Motor:-
Shows torque-speed characteristics of Induction motor.



The induction motor speed can be controlled by varying the stator voltage. This method of speed control is known as stator voltage control. Here the supply frequency is constant.

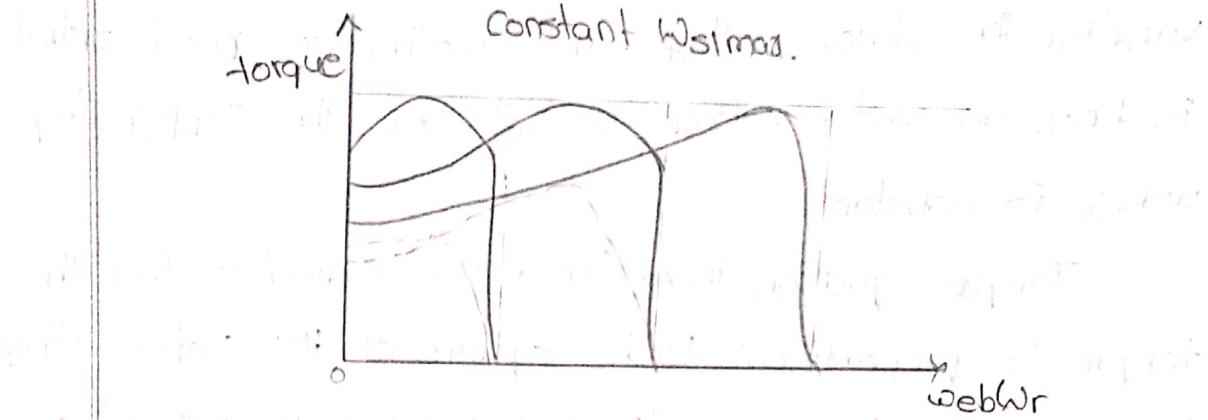
Torque equation of induction motor indicates that the torque is proportional to the square of its stator voltage i.e., $T \propto V^2$. For the same slip and frequency, a small change in stator voltage results in a relatively large change in torque.

The above shows speed-torque characteristics of induction motor under stator voltage control. This characteristic is based on the torque equation. This shows two curves for two different values.

Variable voltage characteristics :-

• Variable frequency control of Induction Motor Drive :- Synchronous speed; therefore the motor speed can be controlled by varying supply frequency. Voltage induced in stator is proportional to the product of supply frequency. Voltage induced in stator is proportional to the product of supply frequency and air-gap flux. If stator drop is neglected, terminal voltage can be considered proportional to the product of frequency and flux.

Any reduction in the supply frequency, without a change in the terminal voltage, causes an increase in the air-gap flux. Induction motors are designed to operate at the knee point of the magnetization characteristic to make full use of the magnetic material.



The above figure variable voltage characteristics.

Variable frequency control of Induction motor by

Voltage Source Inverter:-

A VSI is an inverter in which the dc source has small or negligible impedance. In other words, a VSI has stiff dc voltage source at its input terminals. If viewed from the load side, the ac terminals of the inverter function as a voltage source, i.e., the input voltage should be constant. VSI has low internal function impedance. Because of this, the terminal voltage of a VSI remains constant with variations in load. A VSI is capable of supplying variable frequency and variable voltage for the speed control of induction motor. The forced commutated inverters cannot produce pure sinusoidal waveforms. The motor rotation is smooth even at low frequencies and the motor current is almost sinusoidal in the PWM (pulse width modulation) supply.

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Voltage Source Inverter fed Induction Motor Drive:-

A voltage source inverter (VSI) is an inverter which provides variable voltage and variable frequency from a fixed DC supply. A full bridge VSI is shown in them.

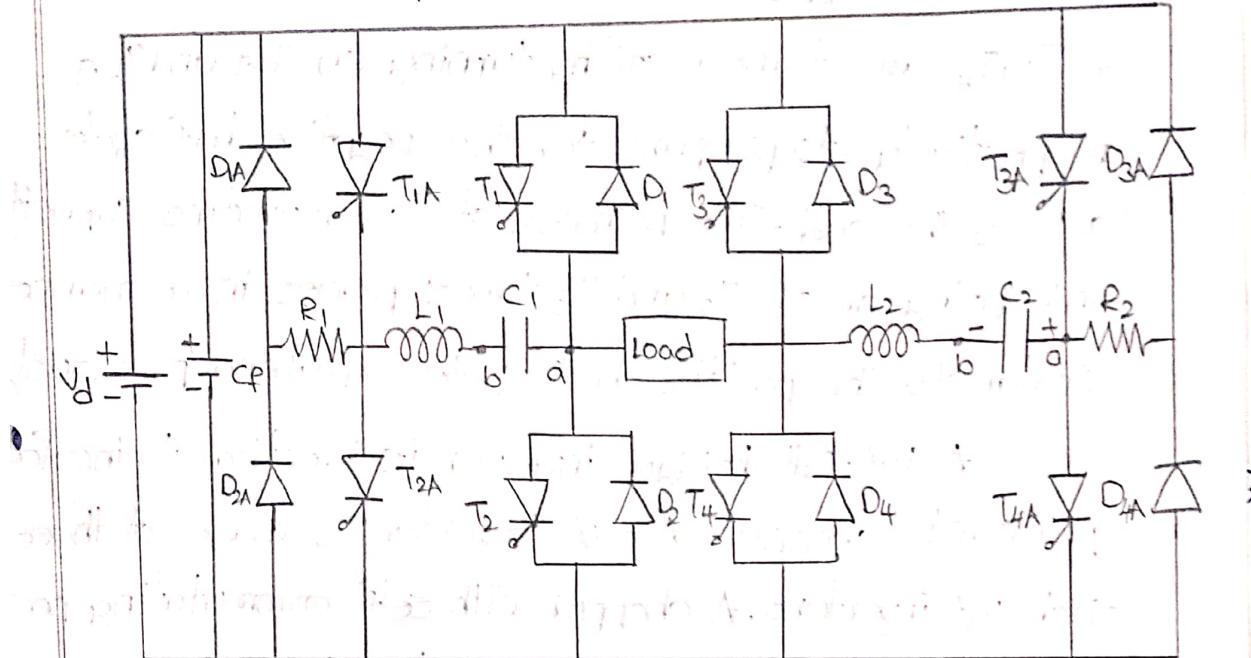


figure:-full bridge voltage source inverter.

The main thyristors are commutated by the auxiliary thyristors, which are denoted by an additional 'A' in their subscripts.

Operation :-

The operation of a VSI consists of the following:

- * T_1, T_2A, T_4 and T_3A are gated on C_1 and C_2 and are charged with 'a' (a positive load voltage) and 'b' (a negative load voltage). The load current are positive and the excess charge in C_1 is drained through V_d , D_{2A} , R_1 , L_1 , C and D_1 . If there is no load current the excess charge in C_2 is drained through V_d , D_4 , L_2 , C_2 , R_2 and D_{3A} .

- * To provide a zero voltage across the load is (shorted)

for part of the positive half cycle, turn T_1 on and thus turn T_4 off. The load is shorted via D_2 , load and T_4 , thus driving the voltage to zero. The zeroing of load voltage is used to charge the capacitors across the load.

* T_1, T_4 are turned off by turning on T_{1A} and T_{4A} respectively, to prepare for the negative half cycle across the load. The primary of commutating capacitors and gating on of T_2 and T_3 takes places in a manner similar to the positive half cycle explained previously.

A 1- ϕ full bridge inverter is identical to the four-quadrant chopper. A 3- ϕ inverter is made of three such 1- ϕ inverters. A chopper with self-commutating power switches such as transistors is shown in fig.

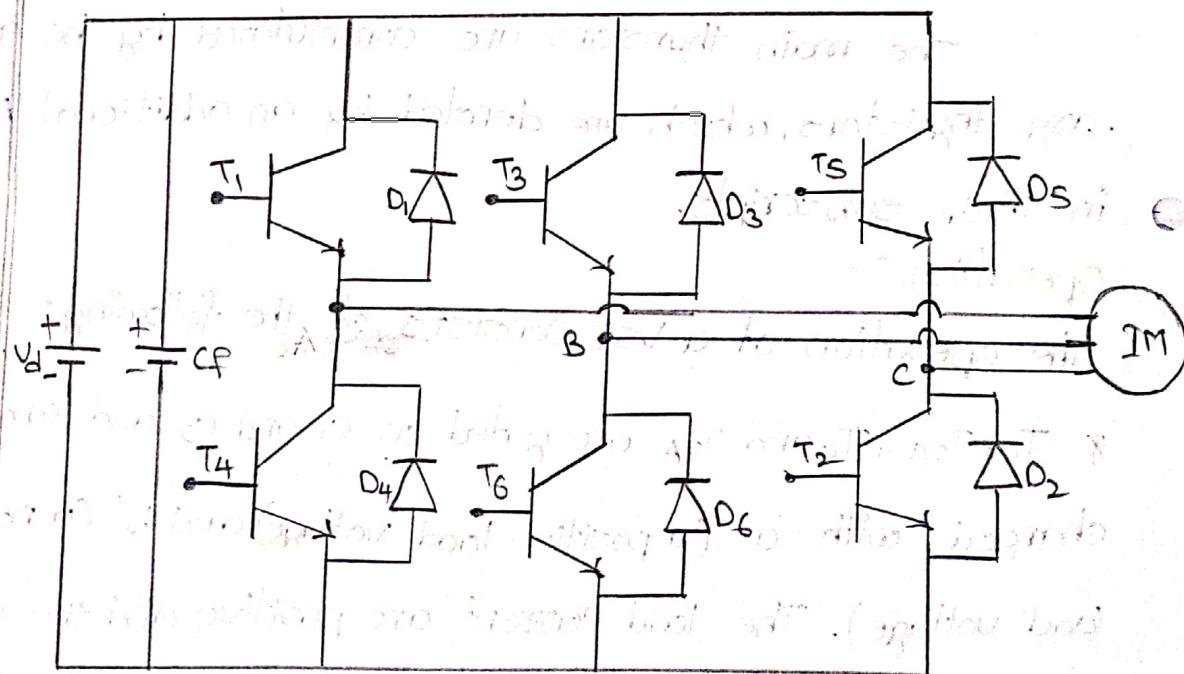


figure :- Voltage source inverter with transistor.

A combination of three of the two quadrant choppers, i.e., with two power switches, results in a 3- ϕ inverter. The voltage available to one phase is always less than the full source or dc link voltage. The auxiliary

(5)

thyristors are not rated to be equal to the main thyristors; they operate for much smaller periods of time compared to the main SCRs. The same argument applies equally to the rating of the auxiliary diodes.

The above VSI is operated inverter and PWM inverter mode. The resultant output waveforms are shown in figure.

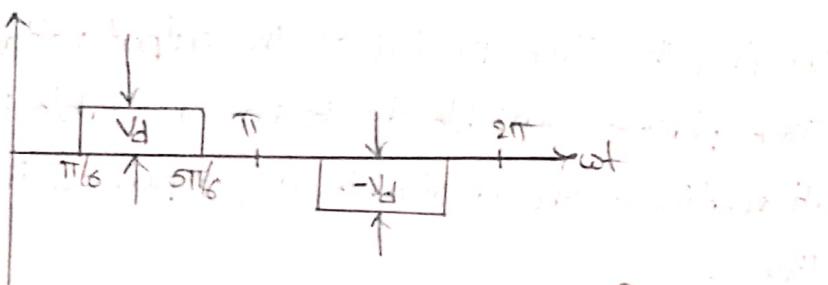


figure :- Line to Line Voltage Waveform.

If the induction motor stator is connected in star manner, then the inverter output line and phase voltages are obtained by applying the Fourier series.

$$V_{AB} = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_d}{n\pi} \cos \frac{n\pi}{6} \sin \left(\omega t + \frac{\pi}{6} \right)$$

$$= \frac{4V_d}{\pi} \cos \left(\frac{\pi}{6} \right) \sin \left(\omega t + \frac{\pi}{6} \right) + \frac{4V_d}{3\pi} \cos \frac{3\pi}{6} \sin 3 \left(\omega t + \frac{\pi}{6} \right) \\ + \frac{4V_d}{5\pi} \cos \frac{5\pi}{6} \sin 5 \left(\omega t + \frac{\pi}{6} \right) + \dots$$

$$V_{AB(L.L)} = \frac{4V_d}{\pi} \left[\cos \left(\frac{\pi}{6} \right) \sin \left(\omega t + \frac{\pi}{6} \right) - \frac{1}{5} \frac{\sqrt{3}}{2} \sin 5\omega t - \frac{1}{7} \frac{\sqrt{3}}{2} \sin 7\omega t + \dots \right] \\ = \frac{4V_d}{\pi} \cdot \frac{\sqrt{3}}{2} \left[\sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t + \dots \right]$$

$$V_{AB(L.L)} = \frac{2\sqrt{3}}{2} \cdot V_d \left[\sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t + \dots \right]$$

RMS value of line to neutral voltage,

$$V_{AN(rms)} = \frac{V_m}{\sqrt{2}} = \frac{2V_d}{\pi} \cdot \frac{1}{\sqrt{2}}$$

$$V_{AN(rms)} = \frac{\sqrt{2}V_d}{\pi}$$

$$V_{AN(rms)} \propto V_d$$

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Line to neutral voltage,

$$V_{AN} = \frac{2V_d}{\pi} \left[\sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t - \dots \right]$$

The variable rms value given to the stator of the induction motor is obtained by varying the dc input voltage, i.e., V_d or V_d and variable frequency is obtained by varying the time period of the output voltage waveform. In some places, variable dc is not available; in such cases alternatives are available as explained in the following lines.

Case (a) :-

When the supply is fixed dc, a variable input dc voltage is obtained by connecting a dc chopper between the dc supply and inverter, because the chopper provides the variable dc output voltage from a fixed dc input voltage as shown in figure.

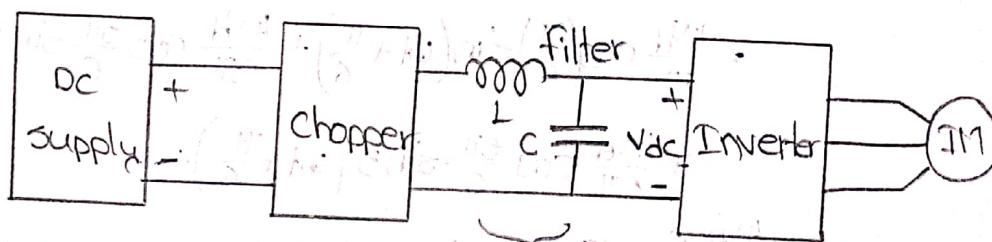
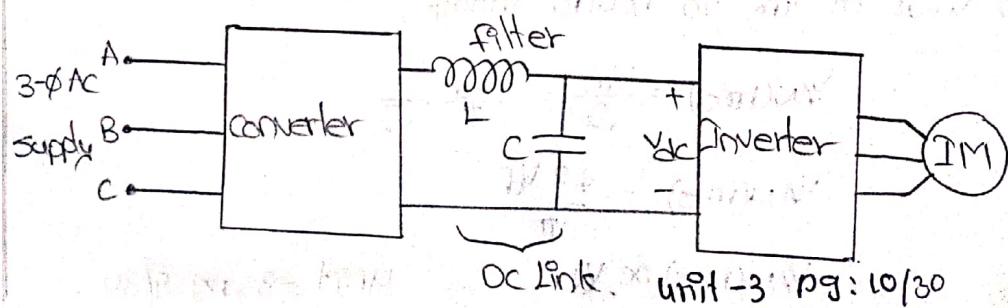


Figure :- Case (a).

Case (b) :-

When the supply is 3-φ ac, the variable dc voltage is obtained by connecting a controlled rectifier i.e., the converter between the 3-φ ac supply and the inverter circuit.

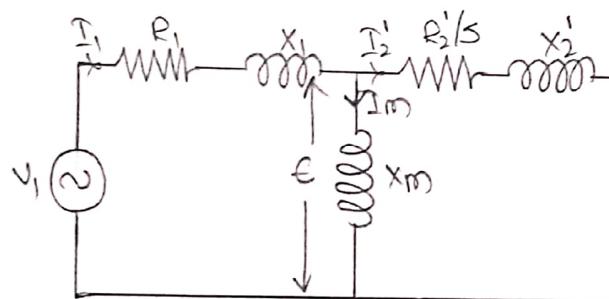


To make the inverter operation independent of the converter of the chopper, a large electrolytic capacitor filter 'c' is connected in dc link and it is also used to filter out the harmonics present in the dc link voltage.

Variable frequency Control from a current source:-

Rotor current, I_2' = Total current $\times \frac{\text{opposite impedance}}{\text{Total impedance of the parallel branches.}}$

$$= I_1 \cdot \frac{x_m}{\sqrt{(R_2' s)^2 + (x_m + x_2')^2}}$$



$$T = \frac{3}{\omega s} I_2'^2 \frac{R_2'}{s}$$

Application of above eqn in the circuit diagram

$$\text{Torque} \propto \frac{3}{\omega s} \frac{I_1'^2 x_m (R_2' s)}{(R_2' s)^2 + (x_m + x_2')^2}$$

For a constant voltage, torque is proportional to $I_1'^2$ and hence torque is proportional to square of stator current.

$$T \propto \frac{I_1'^2 R_2'}{(R_2' s)^2 + (x_m + x_2')^2}$$

From the above eqn, torque is proportional to square of stator current. Hence torque is proportional to square of stator current.

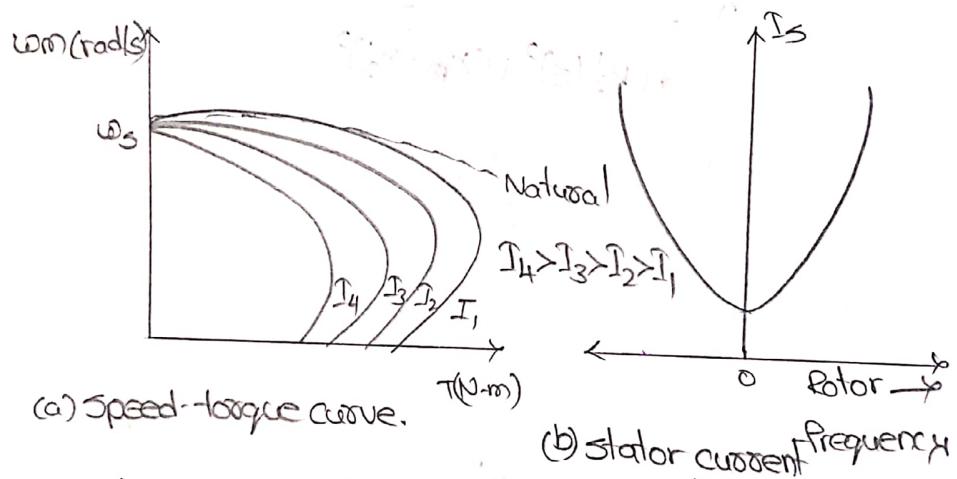
A small change in stator current gives the results of a greater change in developed torque.

The speed-torque characteristics for different values of stator current are shown in figure (i)

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Motor speed-torque curves for various values of I_s and natural speed-torque curves, which

correspond to the operation at rated constant flux, are shown in fig (i); the natural characteristic is locus of preferred operating points from, one obtain a relationship between I_s and rotor frequency (f_r) for rated I_m (or rated flux). This relationship, which is independent of frequency is shown in fig (2). The drive is operated such that a relationship is maintained between stator current I_s and rotor frequency (f_r), when frequency is changed to control the speed.



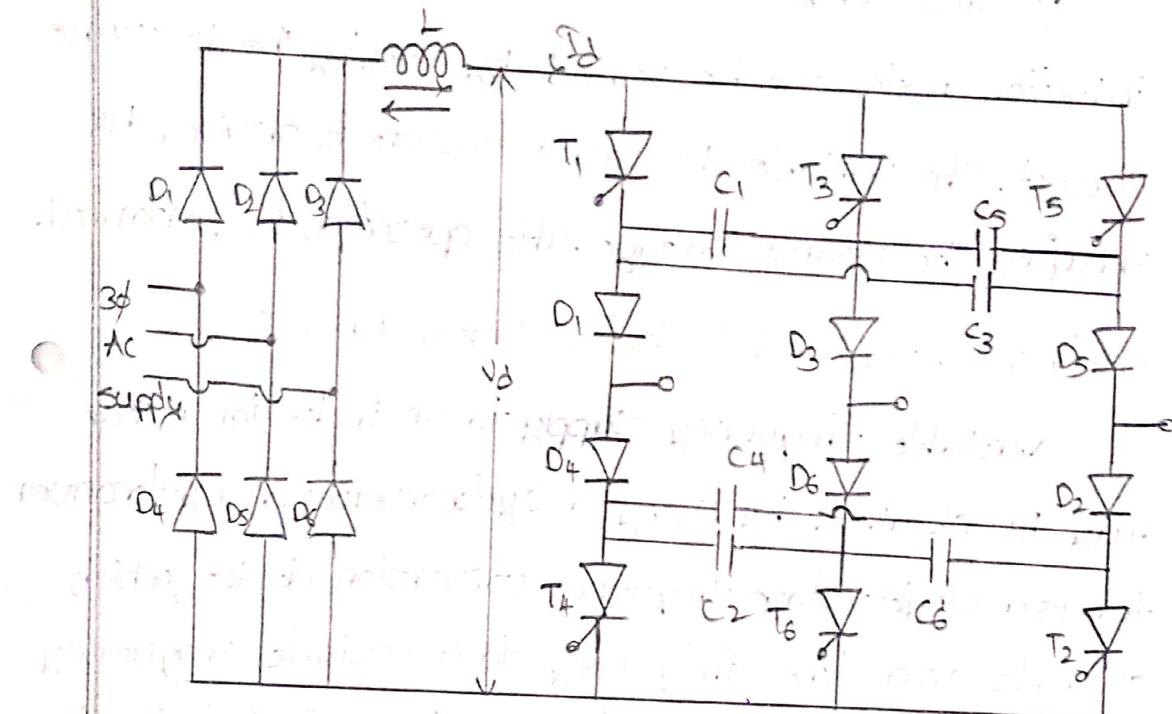
Current Source Inverter :-

In these types of inverters, the controlled quantity is the current in the dc link. The current from the dc source remains constant at the controlled value, irrespective of the load and events in the inverter. The voltage across the load adjust itself. The link current is kept constant by means of a large link inductance L ; the capacitance in the dc link can be dispensed with.

The dc link current is made to flow through the phases of the load alternatively by controlling the inverter. These inverters are classified on the basis of commutation. The control of the link current is achieved by means of phase-controlled rectifier on the line side.

As the current is a controlled quantity, feedback diodes are not required. figure (1) employs individual commutation of phases. Auxiliary thyristors are used for commutation.

The diodes D_1 to D_6 are used to prevent the discharging of capacitors through the load; they thus trap the charge on the capacitors.



(a)

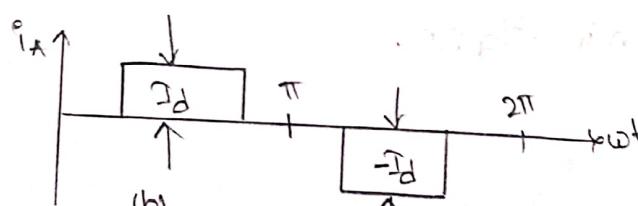


fig :- CSI-fed Induction Motor drive.

Like in the case of the voltage source inverter, Fourier analysis is applied to the current waveform shown in fig(b). It gives the following expression:

$$i_A = \frac{2\sqrt{3}}{\pi} I_{dc} \left[\sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t \dots \right]$$

The fundamental component of motor ac line current has an amplitude of $\frac{2\sqrt{3}}{\pi} I_{dc} = I_m$.

\therefore rms value of the fundamental component,

$$I_{rms} = \frac{I_m}{\sqrt{2}} + \frac{\sqrt{2}\sqrt{3}I_{dc}}{\pi} \times \frac{1}{\sqrt{2}}$$

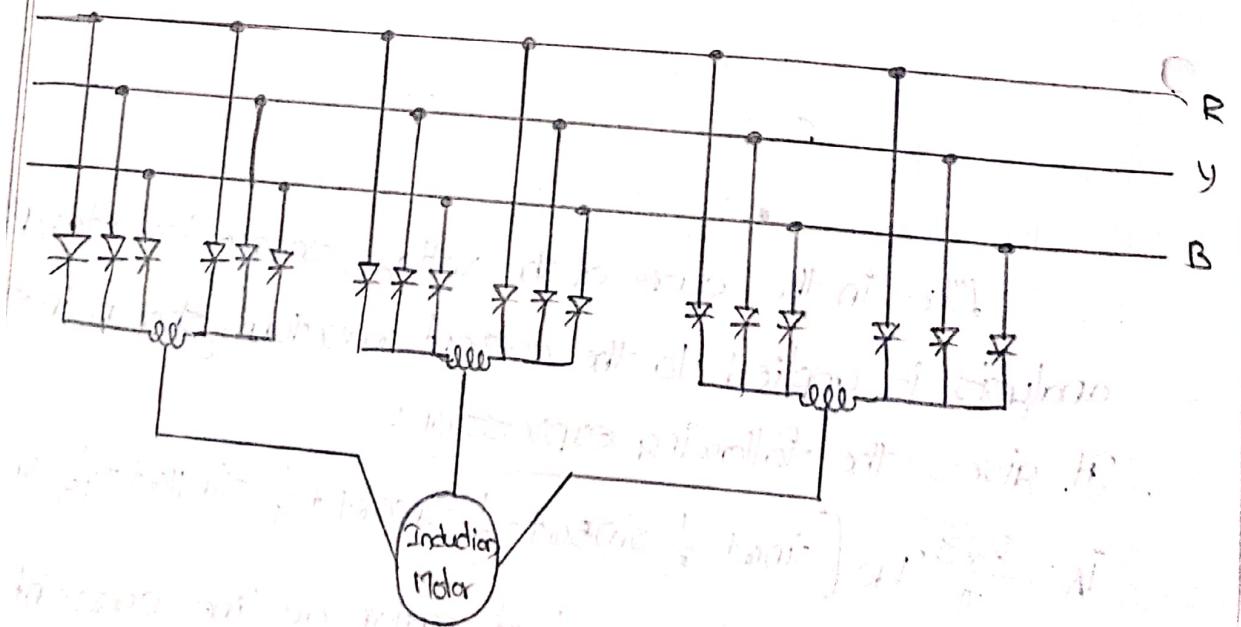
$$I_{rms} = \frac{\sqrt{6}I_{dc}}{\pi}$$

$$I_{rms} \propto I_{dc} \quad [\because \frac{\sqrt{6}}{\pi} \text{ is constant.}]$$

the rms value of current given or supplied to the induction motor can be varied by varying the dc source current. The variable dc source current is obtained by varying the source voltage. This operation is performed.

Cycloconverter fed Induction Motor Drive:-

Variable frequency supply to an induction motor may be obtained by using a cycloconverter. A cycloconverter is a single-stage frequency conversion device which converts an ac line frequency to a variable frequency. A 3-φ, 3-pulse cycloconverter feeding a 3-φ induction motor is shown in figure.



(a) Half wave cycloconverter-fed induction motor.

Half wave cycloconverters are used to obtain variable frequency fed induction motor.

* Control of output voltage and frequency is obtained with variation of firing point of just the controlled rectifier and no change of voltage source.

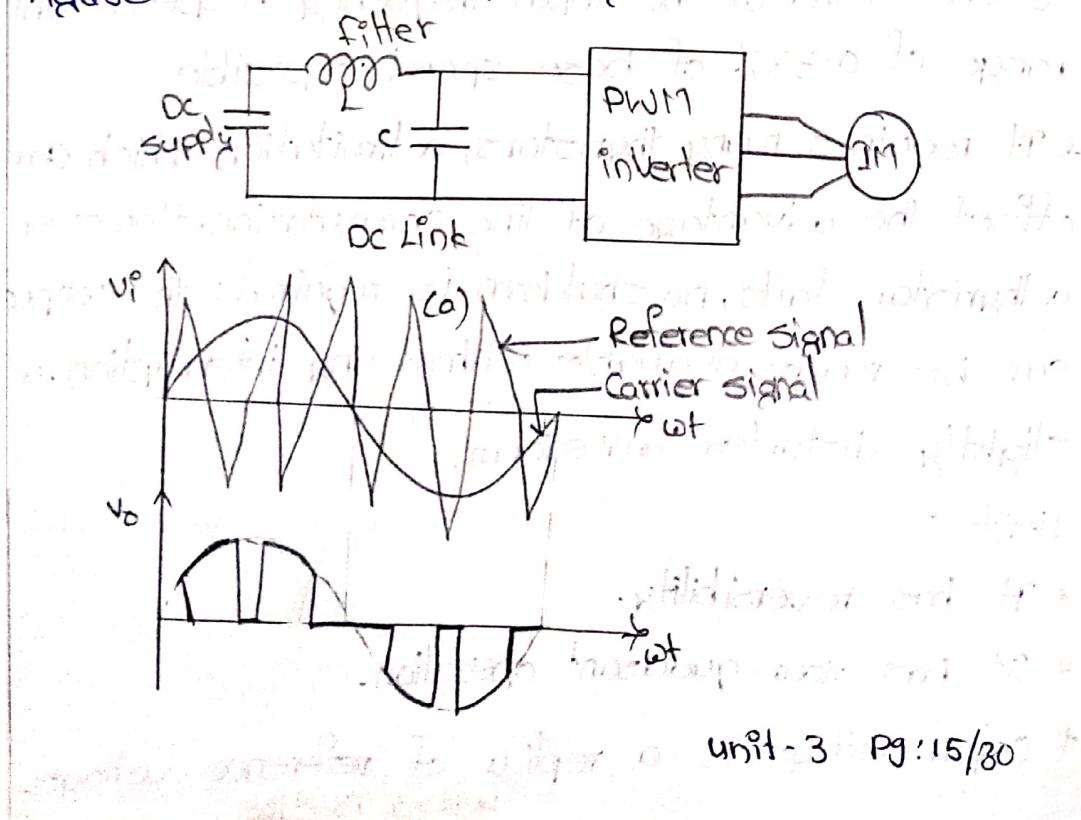
Demerits:-

The demerits of cycloconverters are:

- * It requires a large number of semiconductor devices.
- * There are harmonics and poor power factor at higher ratings.

VSI Operated in PWM Inverter Mode:-

In the stepped wave operation of VSI, referring to eq. the phase to neutral voltage contains harmonics. Due to the presence of these harmonics in the voltage applied to the machine, smooth operation is not obtained. In order to operate the machine smoothly, the VSI must be operated in PWM inverter mode. The relevant diagram is shown in figure and the corresponding waveform shown in figure.



A cycloconverter fed induction motor drive has the

following features:

* Voltage control is possible in the converter itself, so that the machine operates at its rated flux conditions.

* A cycloconverter operates by means of line commutations.

No forced commutation is required as the necessary reactive power for commutation is provided by the line. Losses due to forced commutation can be eliminated. The converter operates at lagging power factor, and the line power factor is very poor at light loads.

* A cycloconverter is capable of power transfer in either direction between an ac source and a motor load.

It can feed power to a load of any power factor. Regeneration is inherent in the complete speed range. A four-quadrant operation is simple and straightforward.

* The output frequency of the cycloconverter is limited to one-third of the input frequency. A speed control range of 0-33% of base speed is possible.

* It requires many thyristors, a limitation which can offset the advantage of line commutation. However, if a thyristor fails, no shutdown is required; the output can be made available without any interruption, with a slightly distorted waveform.

Merits :-

* It has reversibility.

* It has four-quadrant operation.

* Output voltage is a replica of reference voltage.

No additional arrangement is required for the variation of input dc voltage; hence the inverter is directly connected when the input supply is dc. The PWM inverter output voltage can now be controlled by using pulse width modulation.

The function component in the output phase voltage of PWM inverter is given by

$$V = m \frac{V_{dc}}{\sqrt{2}}$$

In this equation $\frac{V_{dc}}{\sqrt{2}}$ = constant. Therefore

Modulation index (m) = constant.

Where m = Modulation index.

Modulation index (m):-

It is defined as the ratio of amplitude of the carrier signal (e_s) to the amplitude of reference signal (e_r),

$$\text{i.e., } m = \frac{\text{Amplitude of } e_s}{\text{Amplitude of } e_r}$$

In this definition, the reference signal amplitude is constant.

$\therefore m \propto$ amplitude of the carrier signal

$$V \propto e_s \quad (\because V \propto m)$$

∴ finally, the output voltage of PWM inverter is controlled by changing the amplitude of the carrier signal.

Suppose that the supply is ac and the PWM inverter is connected through a diode bridge rectifier as shown in figure.

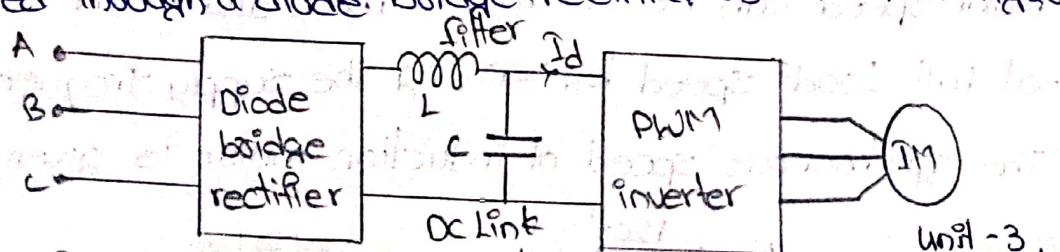


Fig: - PWM inverter connected through diode bridge rectifier.

Comparison of VSI AND CSI DRIVES :-

VSI	CSI
* Input voltage is maintained constant.	* Input current is constant but adjustable.
* The output voltage does not depend on the load.	* The output current does not depend on the load.
* The magnitude of the output current and its waveform depends on the nature of the load impedance.	* The magnitude of the output voltage and its waveform depends on the nature of the load impedance.
* It requires feedback diodes.	* It does not require feedback diodes.
* Commutation circuit is complicated i.e. it contains capacitors and inductors.	* Commutation circuit is simple i.e. it contains only capacitors.

Variable frequency Characteristics :-

The stator frequency control is the one of the speed control is the one of the speed controls of 3- ϕ induction motor. Synchronous speed is directly proportional to supply frequency. Hence, synchronous speed and motor speed can be controlled above and below the normal full load speed by varying the supply frequency.

The synchronous speed of induction motor is given by

$$N_s = \frac{120f}{P}$$

In the above equation, the synchronous speed of the motor is directly proportional to the frequency of the supply voltage. Hence, if the supply frequency is changed the motor speed also changes. But the expression for the air-gap flux is given

$$\phi_g = \frac{1}{4.44 k_1 T_{ph}} \left[\frac{V}{f} \right]$$

Where,

k_1 = stator winding constant.

T_{ph} = stator turns per phase.

Decrease in flux will cause torque capacity increase.

When frequency is decreased below the rated value the voltage is also decreased to retain air gap flux constant and for frequencies above the rated value the voltage is held constant due to induction and supply voltage limitations.

$$k = \frac{f}{f_{rated}}$$

Let

Where

f = operating frequency

k = per unit frequency.

for $k < 1$ (operation below f_{rated}), the motor operates at constant flux.

$$I_m = \frac{E_{rated}}{X_m}$$

Where $X_m \approx 2\pi f_{rated} L_m$.

If frequency is f , then

$$I_m = \frac{C}{2\pi f L_m} = \frac{C}{2\pi k f_{rated} L_m}$$

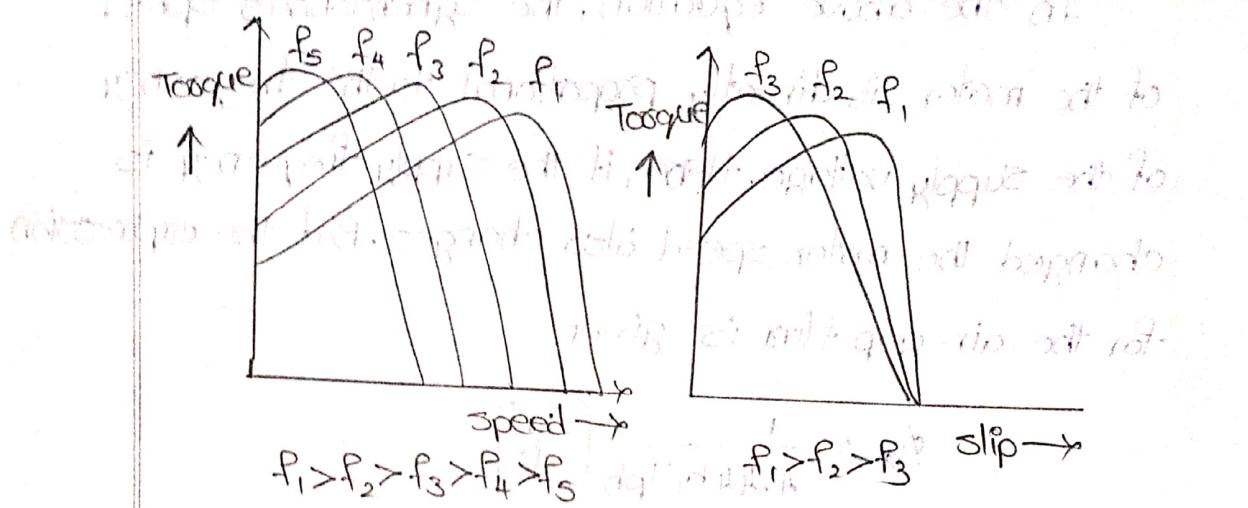


fig:- Speed - torque characteristics of induction motor,
under the condition of input phase reversal.

Closed Loop Speed Control: - ~~motor rotates at different speeds~~

Single quadrant closed loop speed control:

figure shows block diagram of single quadrant closed loop speed control of 3-phase induction motor with stator voltage control.

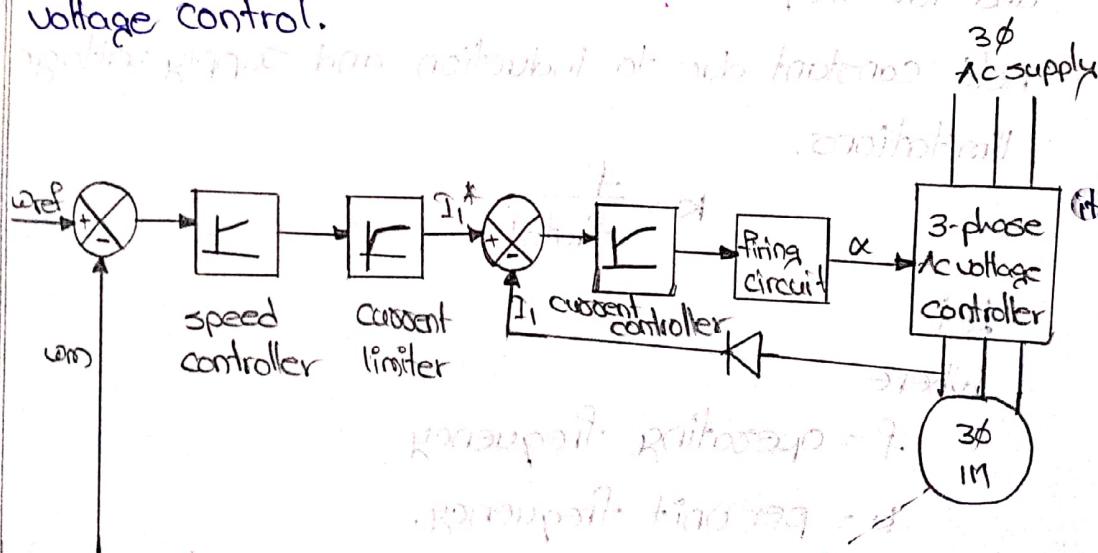


figure :- Single quadrant closed loop speed control.

This block diagram consists of error detector (comparator), speed controller, current limiter, current controller, firing current, current sensing, speed sensing, three phase ac voltage controller and 3 ϕ IM. It consists

$$N_S = \frac{4\pi f}{P} \cdot \frac{10^3}{100}$$

of an inner current loop and outer speed loop.

The induction motor speed can be sensed by using tacho generator. It gives actual motor speed (ω_m). The actual speed is compared with reference speed (ω_{ref}) by using comparator. It gives error signal. This signal fed to the speed controller (P, PI controller). The output of the (speed) Speed controller is fed to current limiter. The current limiter gives reference current of the motor current (I_1). These two currents compared by error detector. The output of the error detector is fed to current controller and firing circuit. The firing circuit generator trigger pulses for 3- ϕ AC voltage controller.

By varying the delay angle of the SCR, the induction motor speed can be controlled.

four quadrant closed loop speed control:

figure shows block diagram four quadrant closed loop speed control using 3- ϕ AC voltage controller. In this method, apart from usual voltage and current controllers an absolute value circuit block and master controller block are also added.

The absolute value circuit gives positive voltage as the output whether the input signal is positive or negative depending on the direction of rotation, so that necessary voltage is developed. The master controller has three input signals and one output signal.

The three informations are:

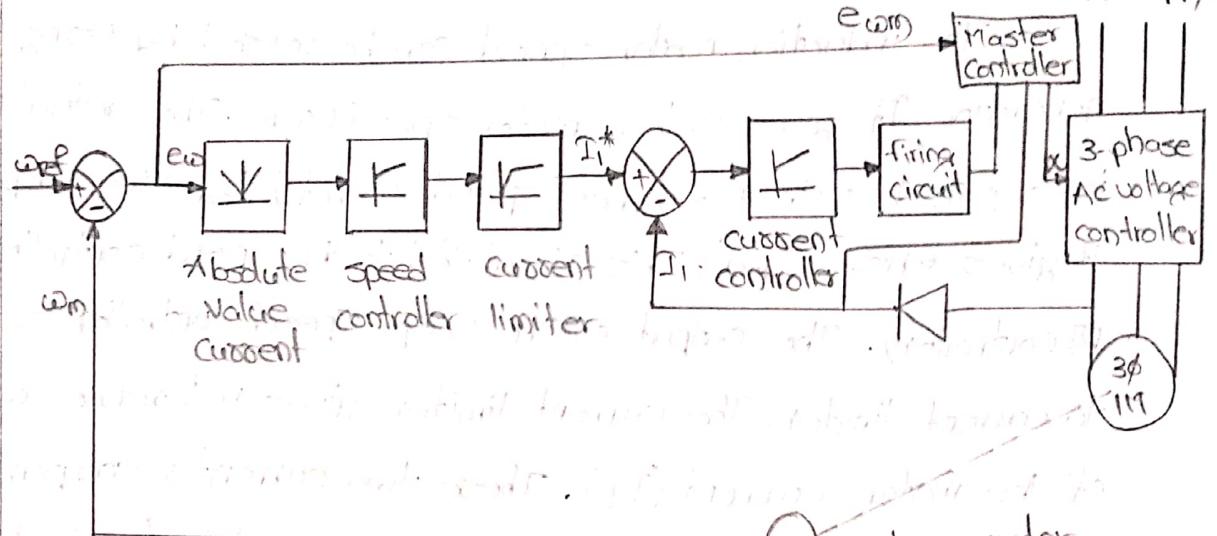


Figure :- Block diagram of four quadrant closed loop speed control.

- 1) The required direction of rotation is given by speed error signal e_w .
- 2) The output signal of the firing circuit.
- 3) Stator current signal.

The master controller receives these three signals and processing these information and give firing pulses to thyristor sets RYB, if positive direction of rotation is to R'YB'. For reverse direction, the firing pulses goes to R'YB'.

Now consider the operation of the speed reversal. When the speed command is set for the reverse direction, the speed error e_w reverses and exceeds a prescribed limit. The speed error signal is given by

$$e_w = \omega_{ref} - \omega_m$$

(12)

When the speed reference command signal ω_{ref} is positive, error signal (e_ω) also positive.

If the speed reference command ω_{ref} is negative, the error signal also negative and very large initially.

$$e_\omega = -\omega_{ref} - \omega_m$$

The master controller senses this large value, withdraws the gate pulses to SCRs sets RYB forcing the current to zero. The master controller provides a delay of 5 to 10 msec after the zero current is sensed, for ensuring that the outgoing SCR's are turned off. Now the gate pulse are now released to other set of SCRs R'YB! The drive first decelerates and accelerates in the other direction at constant maximum allowable current and finally settles at the desired speed.

Problem 5 :-

Example 1 :- A 2.8 kW, 400V, 50Hz, 4 pole, 1370 rpm, delta connected squirrel cage induction motor has the following parameters referred to the stator. $R_s = 2\Omega$, $R_r' = 5\Omega$, $X_s = X_r' = 5\Omega$, $X_m = 80\Omega$. Motor speed is controlled by stator voltage control. When driving a fan load it runs at rated speed at rated voltage. calculate i, Motor terminal voltage, current and torque at 1200 rpm and ii, Motor speed, current and torque for the terminal

Voltage of 300V.

Given data, (from notes) Input voltage, $\text{N}_1 = 1500 \text{ rpm}$.

Power rating of the motor $P = 2.8 \text{ kW}$.

Rated line voltage $(V) = 400V$.

$$f = 50 \text{ Hz}$$

$$P_{\text{input}} = 2.8 \text{ kW} = 2800 \text{ W}$$

$$N_1 = 1500 \text{ rpm.}$$

$$R_s = 2\Omega, R_r' = 2\Omega$$

$$X_s = X_r' = 5\Omega, R_m = 80\Omega$$

$$\text{Speed of motor} N = N_1(1-s)$$

$$\text{and the torque} T = 2.2 \times 1500(1-0.147)$$

$$N = 1279 \text{ rpm}$$

$$\text{current } I_s = I_m + I_r$$

$$I_s = \frac{300}{j80} + \frac{300}{(2+\frac{5}{0.147})+j10}$$

$$= 7.75 - j5.908$$

$$= 9.75 \angle -37.3^\circ \text{ A}$$

$$\text{Line current } I_L = \sqrt{3} \times 9.75$$

$$I_L = 16.88 \text{ A.}$$

Example 2: - A 440V, 3-phase, 50Hz, 6-pole, 945rpm, A-connected induction motor has the following parameters referred to stator: $R_s = 2\Omega, R_r' = 2\Omega, X_s = 3\Omega, X_r' = 4\Omega$

When driving a fan load at rated voltage it runs

at rated speed. The motor speed is controlled by stator voltage control. Determine.

- i, Motor terminal voltage, current and torque at 800rpm
- ii, Motor speed, current and torque for the terminal voltage of 280V.

Given data,

$$V_L = V_{ph} = 440V, f = 50Hz, R_s = 2\Omega, R_r' = 2\Omega, \\ X_s = 3\Omega, X_r' = 4\Omega, N = 945\text{rpm}, P = 6$$

To find.

- i, To determine motor terminal voltage current and torque at 800 rpm.
- ii, To determine motor speed, current and torque for the terminal voltage of 280V.

Solution:

$$i, N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000\text{rpm}.$$

$$\beta = \frac{N_s - N}{N_s} = \frac{1000 - 945}{1000} = 0.055.$$

$$\omega_s = 1000 \times \frac{2\pi}{60} = 104.72 \text{ rad/sec.}$$

$$T = \frac{3}{4\omega^2} \left[\frac{V^2}{(R_s + R_r')^2 + (X_s + X_r')^2} \right] \frac{R_r'}{\beta}.$$

$$= \frac{3}{104.72} \cdot \frac{440^2}{\left(2 + \frac{2}{0.055}\right)^2 + (3+4)^2} \cdot \frac{2}{0.055}.$$

$$T = 132.62 \text{ N-m}$$

$$T = k\omega m^2$$

$$k = \frac{T}{\omega^2 m} = \frac{132.62}{(2\pi \times 945)^2} \times \frac{60}{21}$$

$$k = 0.0135$$

$$T = k\omega m^2 \text{ for } N = 800 \text{ rpm.}$$

$$= 0.0135 \times \left[\frac{2\pi \times 800}{60} \right]^2$$

$$T = 94.75 \text{ N-m}$$

$$S_2 \frac{N_2 - N}{N_2} = \frac{1000 - 800}{1000} = 0.2$$

$$V = I_r' Z_r' \text{ to either terminal.}$$

$$I_r' = \sqrt{\frac{Tk\omega S}{3R_r}}$$
$$= \sqrt{\frac{94.75 \times 104.72 \times 0.2}{3 \times 2}} = 18.19 \text{ A.}$$

$$Z_r' = \sqrt{(R_s + \frac{R_r'}{3})^2 + (X_s + X_r')^2}$$

$$= \sqrt{\left(2 + \frac{2}{0.2}\right)^2 + 7^2} = 13.89 \Omega$$

$$V = I_r' Z_r' = 18.19 \times 13.89 = 252.7 \text{ V.}$$

$$V = 252.7 \text{ V}$$

$$I_r' = 18.19 \text{ A}$$

$$I_L = \sqrt{3} \times 18.19 = 31.51 A$$

$$I_L = 31.51 A$$

$$\text{ii}, \quad \omega_m = \omega_s(1-s)$$

$$\omega_m \approx 104.72 (1-s)$$

$$T^2 = k\omega_m^2$$

$$T = \frac{3}{\omega_s} \left[\frac{\frac{v^2}{(R_s + R_r)^2}}{(R_s + \frac{R_r}{3})^2 + (x_s + x_r)^2} \right] \frac{R_r}{s}$$

$$k\omega_m^2 = \frac{3}{\omega_s} \left[\frac{\frac{v^2}{(R_s + R_r)^2}}{(R_s + \frac{R_r}{3})^2 + (x_s + x_r)^2} \right] \frac{R_r}{s}$$

$$k\omega_m^2 (1-s)^2 = \frac{3}{\omega_s} \frac{v^2}{(R_s + \frac{R_r}{3})^2 + (x_s + x_r)^2} \frac{R_r}{s}$$

$$0.0135 \times 104.72 (1-s)^2 = \frac{3}{104.72} \frac{\frac{280^2}{(2 + \frac{2}{3})^2 + 7^2}}{\frac{2}{3}}$$

$$535^4 - 985^3 + 415^2 - 30.345 + 4 = 0$$

Solving this equation.

$$s = 0.1526$$

$$N = N_s(1-s) = 1000(1-0.1526)$$

$$N = 847 \text{ rpm}$$

$$T = k\omega_m^2$$

$$= 0.0135 \times \left(\frac{2\pi \times 847}{60} \right)^2$$

$$T = 106.2 \text{ N-m}$$

$$I = \frac{V}{Z_r} = \frac{280}{\sqrt{\left(\frac{R_s + R_r}{s}\right)^2 + \left(X_s + X_r\right)^2}}$$

$$= \frac{280}{\sqrt{\left(2 + \frac{2}{0.156}\right)^2 + 7^2}} \approx 16.82 \text{ A.}$$

$$(I_L = \sqrt{3} \times 16.82)$$

$$I_L = 29.13 \text{ A}$$

Example :- (a) A three-phase squirrel cage IM drives a fan-type load. No load rotational losses are negligible. Show that rotor current is maximum when motor runs at a slip $s=1/3$. Find also an expression for maximum rotor current. (b) If three-phase squirrel cage IM runs at speed of n_1 , 1450 rpm and n_2 , 1300 rpm, determine the maximum current in terms of rated current at these speeds. The IM drives a fan and no-load rotational losses are ignored.

Solution.

(a)

The torque required by a fan-type load is proportional to speed squared

$$\therefore T_L = k \omega^2$$

Mechanical power developed in motor, $P_m = (1-s) P_{el}$

(15)

The no-load retentional losses are negligible,

δ_{m1} = power required by load.

(1) $P_{d1} = \delta_{m1}$,

$$1.3V_1^2 \frac{R_1}{3} (1-s)^2 + T_L \omega_m = P_{d1}$$

$$2V_1^2 \frac{R_1}{3} (1-s)^2 + T_L \omega_m$$

$$T_L = \left[\frac{\omega_m T_L \omega_s^2}{3R_1 (1-s)} \right]^{1/2} \quad \rightarrow \textcircled{1}$$

Put $\omega_m = \omega_s (1-s)$ and $T_L = k \omega_m^2$, substituting

these in (1), we get, after simplification,

$$T_L = \left[\frac{\omega_s (1-s) T_L \omega_s^2}{3R_1 (1-s)} \right]^{1/2} = \left[\frac{\omega_s^2 k \omega_m^2}{3R_1} \right]^{1/2}$$

$$= \omega_m \left[\frac{s k \omega_s^2}{3R_1} \right]^{1/2} = \left[\frac{s k \omega_s^2 \omega_m^2}{3R_1} \right]^{1/2}$$

$$= (1-s) \omega_s \left[\frac{s k \omega_s^2}{3R_1} \right]^{1/2}$$

$$T_L = \sqrt{3} (1-s) \left[\frac{k \omega_s^3}{3R_1} \right]^{1/2} \quad \rightarrow \textcircled{2}$$

The slip at which rotor current I_r becomes maximum can be found by taking $\frac{dI_r}{ds}$ and equating it to zero.

$$\frac{dI_r}{ds} = \frac{1}{2} \sqrt{3} (1-s) \left[\frac{k \omega_s^2}{3R_1} \right]^{1/2} + \sqrt{3} (1) \left[\frac{k \omega_s^3}{3R_1} \right] = 0$$

$$(\text{or}) \quad \frac{1-s}{2} = \frac{1}{3}$$

Following, $s = 1/3$ at slip ratio $\rightarrow \text{③}$. Then
 This shows that I_r' is maximum at slip of $1/3$. The
 maximum value of I_r' is obtained by putting $s = 1/3$ in
 equation (2).

$$I_{r\max}' = \sqrt{\frac{1}{3}} \left(1 - \frac{1}{3}\right) \left[\frac{k\omega^3}{3R_f} \right]^{1/2} = \sqrt{\frac{1}{3}} \frac{2}{3} \left[\frac{k\omega^3}{3R_f} \right]^{1/2}$$

$$= \frac{2}{9} \omega_3 \left[\frac{k\omega^3}{R_f} \right]^{1/2} \rightarrow \text{④}$$

(b) from equation (2)

$$I_r' = \sqrt{3} \cdot (1-s) \left[\frac{k\omega^3 s}{3R_f} \right]^{1/2}$$

i, for 1450 rpm, full load slip.

$$s_1 = \frac{1500 - 1450}{1500} = 0.033$$

from above, $\frac{I_{r\max}'}{I_r'} = \frac{\sqrt{\frac{1}{3}} \frac{2}{3}}{\sqrt{3} \cdot (1-s_1)} = \frac{\sqrt{\frac{1}{3}} \frac{2}{3}}{\sqrt{0.83} (1-0.033)} = 2.1919$.

ii, for 1360 rpm, full load slip.

$$s_1 = \frac{1500 - 1360}{1500} = 0.093$$

$$\frac{I_{r\max}'}{I_r'} = \frac{\sqrt{\frac{1}{3}} \frac{2}{3}}{\sqrt{0.093} (1-0.093)}$$

$$\frac{I_{r\max}'}{I_r'} = 1.394$$